Pattern Vectors with the Characteristic Coefficients

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Abstract

Clustering is one of the most important analysis tasks in spatial databases. In this paper, we consider certain pattern vectors arising from coefficients of characteristic polynomial and Ihara function.

Keywords: Characteristic coefficients, Pattern vectors and Clustering

1 Introduction

Graph-based methods play an important role in computer vision and pattern recognition. Although graphs have the powerful capability to represent relational patterns, they are not as easily manipulated as pattern vectors. This is because there is no canonical ordering of the nodes of a graph. For this reason, much research has been devoted to the node correspondence problem and pattern matching[2, 6].

There have been a number of recent applications of zeta functions in computer vision and pattern recognition. Zhao et al. [13] have used Savchenko's formulation of the zeta function, expressed in terms of cycles, to generate merge weights for clustering over a graph-based representation of pairwise similarity data [13]. Bai et al. [1] have shown that the Riemann zeta function is the moment generating function of the heat-kernel trace and have used the moments to cluster graphs. Luo et al. showed how to use the Laplacian spectrum as a permutation invariant to cluster graphs [8]. Wilson in [12] showed that the Laplacian spectra give a competitive performance in clustering graphs over various alternative methods [12]. In [10] Ren et al. proposed a method based on Ihara coefficients for obtaining pattern vectors for objects on COIL dataset, which is a standard benchmark in pattern recognition, and then they used K-means algorithm to obtain clusters.

The aim of this paper is to develop a method based on characteristic polynomial of graphs for characterizing graph-based representations. Indeed, using K-means clustering method, we categorize the observed data and compare the obtained results with the proposed method in [10], and determine the effectiveness of our method.

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2 Basic definitions

In this section we summarize some known concepts and facts related to our work.

Definition 1 *The characteristic polynomial of a graph G with n vertices and the adjacency matrix A is defined as*

$$\mathcal{C}(G,t) = \det(t\boldsymbol{I} - A) = t^n + c_1 t^{n-1} + \cdots + c_{n-1}t + c_n$$

where $\boldsymbol{I}$ is the identity matrix of size n.

The set of coefficients of the characteristic polynomial can be applied to characterize the graph structure, for example c_3 is related to the number of triangles and c_4 is related to the difference of the number of 2-matchings and twice the number of 4-cycles in a simple graph G, respectively, see for example [4, 5].

Theorem 1 *(Bass [3]) Let $G = (V, E)$ be a graph with $|V| = n$ vertices and $|E| = m$ edges. Then the Ihara zeta function of G satisfies*

$$\begin{aligned} Z_G^{-1}(u) &= (1-u^2)^{m-n}\det(\boldsymbol{I} - Au + Qu^2) \\ &= u^{2m} + c_1 u^{2m-1} + \cdots + c_{2m} \end{aligned}$$

where A is the adjacency matrix of G and $D = diag(d_1, \dots, d_n)$ is the degree matrix of G which is the diagonal matrix where d_i is the degree of the vertex i in G and $Q = D - \boldsymbol{I}$.

The set of coefficients of the Ihara zeta function, like coefficients of the characteristic polynomial, expresses some information about the structure of the graph see [11] for example.

3 Our method and experimental evaluation

In this section, we construct pattern vectors from coefficients of the characteristic polynomial of graphs. The set $A = \{c_0, c_1, c_2, \dots, c_n\}$ from equation 1 consists of the coefficients of characteristic polynomial. We choose certain subsets of A as pattern vectors for clustering different objects.

Figure 1: COIL Objects and their Delaunay Triangulations

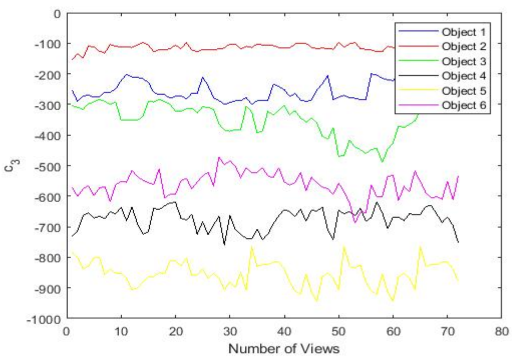

Figure 2: The coefficient c_3 for 72 different views of objects in Figure 1

For this purpose, we use the objects in the COIL dataset with different views [9], and we use Harris corner detector [7] which effectively identifies key structures to construct graphs over these objects and we construct a Delaunay triangulation based on certain feature points as vertices. Figure 1 shows examples of COIL objects and their corresponding Delaunay triangulation.

The diagram in Figure 2 clearly shows that the coefficient c_3 from the characteristic polynomial alone can separate six objects in Figure 1.

As mentioned before, since coefficients c_3 and c_4 of the characteristic polynomial are related to the number of triangles, squares and 2-matchings in the graph, we select the subset $A_s = \{c_3, c_4\}$ of A as a pattern vector of each object. Then we apply K-means algorithm to

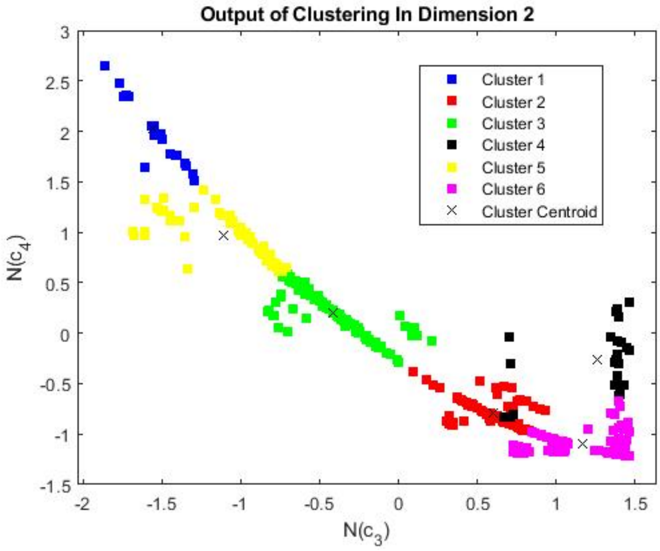

Figure 3: Output of Clustering of our Objects.

Pattern vector	Rand Index
Ihara coefficients	0.95
Characteristic coefficients	0.97

Table 1: Rand Indices from Objects in K-means Method

cluster the objects based on the pattern vectors in the 2-dimensional space. To evaluate the method, the rand index value is calculated, see Table 1. The clustering result is shown in Figure 3. In this figure the normalized coefficients c_3 and c_4 are shown with $N(c_3)$ and $N(c_4)$. The number of a cluster in Figure 3 comes from the corresponding object from Figure 1, from left to right.

4 Conclusion

We used coefficients of the characteristic polynomial of graphs to construct pattern vectors for clustering objects. Experiments were conducted on real-world objects and after calculating the rand indices, the effectiveness of the pattern vectors including characteristic polynomial coefficients compared to the Ihara coefficients was determined.

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